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On a Weaker Notion of Controllability of a Language K with respect to a Language L

Ramavarapu S. Sreenivas

Abstract—**A prefix-closed language $K \subseteq \Sigma^*$ is said to be controllable with respect to another prefix-closed language $L \subseteq \Sigma^*$ if and only if i) $K \subseteq L$, and ii) $K\Sigma_u \cap L \subseteq K$, where $\Sigma = \Sigma_u \cup \Sigma_c$ and $\Sigma_u \cap \Sigma_c = \varnothing$ (cf. [6]). In this note, we consider a weaker notion of controllability where it is not required that $K \subseteq L$. If L is the prefix-closed language generated by a plant automaton G, then essentially there exists a supervisor Θ that is complete with respect to G such that $L(\Theta|G) = K \cap L$ if and only if K is weakly controllable with respect to L (cf. [6, proposition 5.1]). For an arbitrary modeling formalism we show that the inclusion problem is reducible to the problem of deciding the weaker notion of controllability. Therefore, removing the requirement that $K \subseteq L$ from the original definition of controllability does not help the situation from a decidability viewpoint. This observation is then used to identify modeling formalisms that are not viable for supervisory control of the untimed behaviors of discrete event dynamic systems.**

I. Introduction

We assume familiarity with supervisory control of the untimed behaviors of *discrete event dynamic systems* (DEDS). For a detailed treatment of this material the reader is referred to the original papers by Ramadge and Wonham [6], [5]. In particular, we consider the *forbidden string problem* [6], where the objective is to synthesize a supervisor that prevents the occurrence of certain strings in the closed-loop behavior.

In reference [6], the *controllability* of a prefix-closed language $K \subseteq \Sigma^*$ with respect to another prefix-closed language $L \subseteq \Sigma^*$ is defined as i) $K \subseteq L$, and ii) $K\Sigma_u \cap L \subseteq K$, where $\Sigma = \Sigma_u \cup \Sigma_c$ and $\Sigma_u \cap \Sigma_c = \varnothing$. If L is the prefix-closed language generated by a plant automaton G, then the controllability of K with respect to L is a necessary and sufficient condition for the existence of a supervisor Θ that is complete with respect to G such that $L(\Theta|G) = K \cap L = K$ (cf. [6, proposition 5.1]). We call the reader's attention to the fact that if the requirement $K \subseteq L$ is relaxed in the definition of controllability the above result essentially remains unaltered. The only difference is that $L(\Theta|G)$ is no longer the language K but instead the language $K \cap L$. In this note, we define the weak controllability of K with respect to L as the satisfaction of $K\Sigma_u \cap L \subseteq K$.

Using the decidability of weak controllability as the test for viability of modeling formalisms it was thought that weak controllability might result in a larger set of viable modeling formalisms as compared to those obtained for the usual definition of controllability [7]. In this note, we show that the problem of deciding $L \subseteq K$ is reducible to the problem of deciding the weak controllability of K with respect to L. This observation implies that modeling formalisms where the *inclusion problem* is undecidable are unsuitable for representing the untimed behaviors of DEDS. Also, this implies that removing the requirement of $K \subseteq L$ from the original definition of controllability does not help the situation from a decidability viewpoint.

Manuscript received April 17, 1992; revised June 5, 1992 and August 7, 1992. This work was supported in part by the National Science Foundation under Grant CDR-8803012, ECS-91-02346, in part by the Office of Naval Research Grants N00014-90-J-1093 and N00014-89-J-1023 and in part by the Army Grant DAAL-03-91-G-0194 and DAAL-03-86-K-0171.

R. S. Sreenivas is with the Pierce Hall G12h, Division of Applied Sciences, Harvard University, Cambridge, MA 02138.

IEEE Log Number 9208707.

II. Main Result

A prefix-closed language $K \subseteq \Sigma^*$ is said to be *weakly controllable* with respect to another prefix-closed language $L \subseteq \Sigma^*$ if and only if $K\Sigma_u \cap L \subseteq K$. For an arbitrary modeling formalism the inclusion problem is reducible to the problem of deciding controllability. To see this, observe that when $\Sigma_u = \varnothing$, K is controllable with respect to L if and only if $K \subseteq L$. Hence, undecidability of inclusion implies undecidability of controllability. In reference [7], it was suggested that dropping the requirement that $K \subseteq L$ might result in a larger set of viable formalisms for supervisory control. Theorem 2.1 shows that this is not the case as the inclusion problem reduces to the problem of deciding weak controllability.

Theorem 2.1: For an arbitrary symbol set Σ that can be partitioned as $\Sigma = \Sigma_u \cup \Sigma_c$ and any two prefix-closed languages $K, L \subseteq \Sigma^*$, $K \neq \varnothing$ if there exists a procedure to decide $K\Sigma_u \cap L \subseteq K$ then it is possible to decide if $L \subseteq K$.

Proof: For each $\sigma \in \Sigma$ define a partition Π_σ of $\Sigma = \Sigma_u \cup \Sigma_c$, where $\Sigma_u = \{\sigma\}$ and $\Sigma_c = \Sigma - \Sigma_u$.

We claim,

$$(\forall \Pi_\sigma, K\Sigma_u \cap L \subseteq K) \Leftrightarrow L \subseteq K. \tag{1}$$

We first establish $(\forall \Pi_\sigma, K\Sigma_u \cap L \subseteq K) \Rightarrow L \subseteq K$ by contradiction. Let $(\forall \Pi_\sigma, K\Sigma_u \cap L \subseteq K)$ and $L \subsetneq K$. Since L and K are prefix-closed, $K \neq \varnothing$ and $L \subsetneq K$, $\exists \omega \in \Sigma^*$, $\exists \hat{\sigma} \in \Sigma$, such that $\omega \in K \cap L$, $\omega\hat{\sigma} \in L$ and $\omega\hat{\sigma} \notin K$. But this would imply that for the partition $\Pi_{\hat{\sigma}}$, $K\Sigma_u \cap L \subsetneq K$. A contradiction.

We now establish $L \subseteq K \Rightarrow (\forall \Pi_\sigma, K\Sigma_u \cap L \subseteq K)$ by contradiction. Let $L \subseteq K$ and $\exists\ \Pi_{\hat{\sigma}}$ such that $K\Sigma_u \cap L \subsetneq K$. Since L and K are prefix-closed and $K\Sigma_u \cap L \subsetneq K$, where $\Sigma_u = \{\hat{\sigma}\}$, $\exists \omega \in K \cap L$ such that $\omega\hat{\sigma} \in L$ and $\omega\hat{\sigma} \notin K$. But this implies that $L \subsetneq K$. A contradiction. Hence, the result. ♣

Corollary 2.1: The undecidability of inclusion in any modeling formalism implies the undecidability of weak controllability (and also, the undecidability of controllability).

As a consequence of the above corollary we note that removing the requirement of $K \subseteq L$ from the original definition of controllability does not help the situation from a decidability viewpoint. Also, we observe that some of the well-known modeling formalisms are unsuitable for supervisory control of untimed behaviors of DEDS. The unsuitable formalisms include deterministic context-free grammars, context-free grammars, context-sensitive grammars (cf. [2, Fig. 11.5, ch. 11]), Turing machines (cf. [2, Rice's theorem, ch. 8]) and unrestricted Petri nets (cf. [1, theorem 7.3]).

In formalisms where the inclusion problem is decidable some additional work is required to establish the decidability of controllability or weak controllability. These include Finite-state Automata, Free-labeled Petri nets (cf. [3, ch. 6]) and Deterministic-labeled Petri nets (cf. [8]). The controllability and weak controllability of a prefix-closed language K with respect to another prefix-closed language L represented in these formalisms is known to be decidable (cf. [4], [7]).

III. Conclusions

Ramadge and Wonham [6] define the *controllability* of a prefix-closed language $K \subseteq \Sigma^*$ with respect to another prefix-closed language $L \subseteq \Sigma^*$ as the satisfaction of the following conditions i) $K \subseteq L$, and ii) $K\Sigma_u \cap L \subseteq K$, where the symbol set Σ is partitioned as $\Sigma = \Sigma_u \cup \Sigma_c$. They also show that if L is the prefix-closed behavior of a plant automaton G, then the controllability of K with respect to L is necessary and sufficient for the existence of a supervisor Θ that is complete with respect to G such that $L(\Theta|G) = K \cap L = K$. In this note, we consider a weaker notion of controllability where it is not required that $K \subseteq L$. The weaker notion of controllability is necessary and sufficient for the existence of a supervisor Θ that is complete with respect to G such that $L(\Theta|G) = K \cap L$. We note that the inclusion problem reduces to the weaker notion of controllability. This observation is then used to identify modeling formalisms that are unsuitable for supervisory control of untimed behaviors of DEDS. Also, this implies that removing the requirement of $K \subseteq L$ from the original definition of controllability does not help the situation from a decidability viewpoint.

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A Novel Proof of the Souriau-Frame-Faddeev Algorithm

Mao-Da Tong and Wai-Kai Chen

Abstract—**A simple and elegant proof is given for a recurrence relation that computes the coefficients of the characteristic polynomial of a linear system used in the Souriau-Frame-Faddeev algorithm.**

Manuscript received March 27, 1992.

M.-D. Tong is with the Department of Automation, University of Science and Technology of China, Hefei, AnHui 230027 and is currently a visiting scholar at the Department of Electrical Engineering and Computer Science, University of Illinois at Chicago, Chicago, IL 60607.

W.-K. Chen is with the Department of Electrical Engineering and Computer Science, University of Illinois at Chicago, Chicago, IL 60607.

IEEE Log Number 9208688.

The classical Souriau-Frame-Faddeev algorithm is very useful in computing the resolvent matrix used in the solution of the state equation. The algorithm has been extensively covered in most of the books on linear system theory [1]–[18]. However, the majority of them do not give a complete proof with the exception of two [1], [2]. Unfortunately, all these proofs are complicated [3], [4]. In this note we present a new and elegant proof of this algorithm.

Suppose that a linear system is described by its system matrix $\mathbf{A}$ of order n. Its corresponding resolvent matrix $(s\mathbf{1}_n - \mathbf{A})^{-1}$ is denoted as follows:

$$(s\mathbf{1}_n - \mathbf{A})^{-1} = \frac{\operatorname{adj}(s\mathbf{1}_n - \mathbf{A})}{\det(s\mathbf{1}_n - \mathbf{A})} = \frac{\sum_{k=0}^{n-1} \mathbf{Q}_k s^k}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{1}$$

where $\mathbf{Q}_k$, $k = 0, 1, \cdots, n-1$, are coefficient matrices of order n.

The Souriau-Frame-Faddeev algorithm indicates that a_k and $\mathbf{Q}_k$, $k = 0, 1, \cdots, n-1$, can be obtained by the following recurrence formulas:

$$\mathbf{Q}_{n-1} = \mathbf{1}_n \tag{2}$$

$$a_k = \frac{1}{k}\operatorname{tr}\mathbf{Q}_{k-1} = -\frac{1}{n-k}\operatorname{tr}(\mathbf{A}\mathbf{Q}_k) = -\frac{1}{n-k}\operatorname{tr}(\mathbf{Q}_k\mathbf{A}) \tag{3}$$

$$\mathbf{Q}_{k-1} = \mathbf{Q}_k\mathbf{A} + a_k\mathbf{1}_n = \mathbf{A}\mathbf{Q}_k + a_k\mathbf{1}_n, \quad k = n-1, n-2, \cdots, 1 \tag{4}$$

$$a_0 = -\frac{1}{n}\operatorname{tr}(\mathbf{A}\mathbf{Q}_0) = -\frac{1}{n}\operatorname{tr}(\mathbf{Q}_0\mathbf{A}), \quad \text{or} \quad \mathbf{0} = \mathbf{Q}_0\mathbf{A} + a_0\mathbf{1}_n \tag{5}$$

where tr stands for the trace of a matrix. The last equation can be used as a check.

The proofs of (2), (4), and (5) are similar to those given in [1]–[4], and are omitted. The proof of (3) is usually complicated, and a simple novel proof is given below:

$$\begin{aligned}\frac{d}{ds}[\det(s\mathbf{1}_n - \mathbf{A})] &= \frac{d}{ds}\det[\delta_{ij}s - a_{ij}] \\ &= \sum_{i=1}^{n} \text{cofactor of the } (i,i)\text{th element of } (s\mathbf{1}_n - \mathbf{A}) \\ &= \sum_{k=0}^{n-1}(\operatorname{tr}\mathbf{Q}_k)s^k \\ &= ns^{n-1} + (n-1)a_{n-1}s^{n-2} + \cdots + 2a_2 s + a_1\end{aligned} \tag{6a}$$

where $\mathbf{A} = [a_{ij}]$ and

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \qquad i, j = 1, 2, \cdots, n. \tag{6b}$$