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Some observations on supervisory policies that enforce liveness in partially controlled Free-Choice Petri nets☆

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Abstract

Every arc from a place to a transition in a *Free-Choice Petri net* (FCPN) is either the unique output arc of the place, or, the unique input arc to the transition [M.H.T. Hack, Analysis of production schemata by Petri nets, Master's thesis, Massachusetts Institute of Technology, February 1972; W. Reisig, Petri Nets, Springer-Verlag, Berlin, 1985; T. Murata, Petri nets: properties, analysis and applications, Proc. IEEE 77 (4) (1989) 541–580]. We consider FCPNs that are not *live* [J.L. Peterson, Petri Net Theory and the Modeling of Systems, Prentice-Hall, Englewood Cliffs, NJ, 1981; W. Reisig, Petri Nets, Springer-Verlag, Berlin, 1985; T. Murata, Petri nets: properties, analysis and applications, Proc. IEEE 77 (4) (1989) 541–580], and we investigate the existence of supervisory policies that can enforce liveness in *partially controlled* FCPNs. The external agent, or supervisor, can only prevent the firing of some (i.e. not all) transitions in a partially controlled FCPN.

We first present an observation on supervisory policies that enforce liveness in partially-controlled FCPNs. Using this observation, we solve the supervisory synthesis problem for the family of *choice-controlled* FCPNs, defined in this paper. We then identify a new, sub-class of partially-controlled FCPNs that posses an easily-characterized (and easily-enforced) supervisory policy that enforces liveness.

Keywords: Supervisory control; Petri nets

1. Introduction

A large class of systems can be modeled as systems with independent, interacting, concurrent components. Typically, each independent process is split into several operations; the execution of each operation is conditioned on the satisfaction of a set of logical preconditions. Upon the execution of any such operation, a new set of logical conditions is created that inhibit the execution of some operations and enables the execution of others in the system. The supervisory control of such systems requires an external agent to regulate, or limit, the operations of each component so as to guarantee a common objective. In this paper we concern ourselves with a stronger version of deadlock avoidance called *liveness*. From any reachable state of a *live* system, it should be possible for any of the components to execute any of its operations, although not necessarily immediately.

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Petri nets (PNs) [11,12] are an ideal choice of the modeling of such systems as they allow easy representation of the logical preconditions as the marking, and the operations can be represented as transitions. A PN is said to be *live*, if from any reachable marking it is possible to fire any transition, although not necessarily immediately. A *Free-Choice Petri net* (FCPN) is a restricted class of Petri nets where every arc from a place to a transition is either the unique output arc from that place, or, the unique input arc to the transition. In this paper we consider *partially controlled* FCPNs where the external agent, the supervisor, can prevent only some (i.e. not all) transitions from firing. We concern ourselves with the synthesis of supervisory policies that enforce liveness in non-live, partially controlled FCPNs.

We show that a minimally restrictive supervisory policy that enforces liveness in a partially controlled FCPN does not prevent the firing of transitions t that satisfy the property $({}^{\bullet}t)^{\bullet} = \{t\}$. This observation is used to show that it is possible to decide if there is a liveness enforcing policy for a restricted class of FCPNs called *choice-controlled* FCPNs. A choice-controlled FCPN has the property that all uncontrollable transitions satisfy the condition $({}^{\bullet}t)^{\bullet} = \{t\}$. This is followed by the identification of a new class of (not necessarily choice-controlled) partially-controlled FCPNs that have an easily characterizable (and easily enforceable) supervisory policy that enforces liveness.

Section 1 contains the notations and definitions used in this paper. The main results are presented in Section 4 after a review of the relevant results in the literature in Section 3. The paper concludes with Section 5, where the main results of the paper are presented in précis, along with some suggested future research directions.

2. Notations and definitions

A *Petri net* $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is an ordered 4-tuple, where $\Pi = \{p_1, \ldots, p_n\}$ is a set of *n places*, $T = \{t_1, \ldots, t_m\}$ is a collection of *m transitions*, $\Phi \subseteq (\Pi \times T) \cup (T \times \Pi)$ is a set of *arcs*, $\mathbf{m}^0 : \Pi \to \mathcal{N}$ is the *initial marking function* (or the *initial marking*), and $\mathcal{N}$ is the set of non-negative integers. The state of a PN is given by the *marking* $\mathbf{m} : \Pi \to \mathcal{N}$ which indicates the distribution of *tokens* in each place.

For a given marking $\mathbf{m}$, a transition $t \in T$ is said to be *enabled* if $\forall p \in ({}^{\bullet}t)_N$, $\mathbf{m}(p) \geq 1$, where $({}^{\bullet}x)_N := \{y \mid (y, x) \in \Phi\}$. For a given marking $\mathbf{m}$ the set of enabled transitions is denoted by the symbol $T_{\mathrm{e}}(N, \mathbf{m})$. An enabled transition $t \in T_{\mathrm{e}}(N, \mathbf{m})$ can *fire*, which changes the marking $\mathbf{m}$ to $\hat{\mathbf{m}}$ according to the equation

$$\hat{\mathbf{m}}(p) = \mathbf{m}(p) - card((p^{\bullet})_N \cap \{t\}) + card(({}^{\bullet}p)_N \cap \{t\}),$$

where $(x^{\bullet})_N := \{y \mid (x, y) \in \Phi\}$ and the symbol $card(\bullet)$ is used to denote the cardinality of the set argument. A similar notation is used to denote the predecessor or successor set of a set of places or transitions. Sometimes $(x^{\bullet})_N$, (or, $({}^{\bullet}x)_N$) is just represented as $x^{\bullet}$ (${}^{\bullet}x$), when there is no confusion as to the identity of PN N in its definition. In this paper we do not consider simultaneous firing of multiple transitions.

A collection of places $P \subseteq \Pi$ is said to be a *siphon* (*trap*) if ${}^{\bullet}P \subseteq P^{\bullet}$ ($P^{\bullet} \subseteq^{\bullet} P$). A trap (siphon) P, is said to be *minimal* if $\nexists \tilde{P} \subset P$, such that $\tilde{P}^{\bullet} \subseteq^{\bullet} \tilde{P}$ (${}^{\bullet}\tilde{P} \subseteq \tilde{P}^{\bullet}$).

A string of transitions $\omega = t_1 t_2 \ldots t_k$, where $t_i \in T$ ($i \in \{1, 2, \ldots, k\}$) is said to be a *valid firing sequence* starting from the marking $\mathbf{m}$, if

(1) the transition t_1 is enabled under the marking $\mathbf{m}$, and
(2) for $i \in \{1, 2, \ldots, k-1\}$ the firing of the transition t_i produces a marking under which the transition t_{i+1} is enabled.

If $\mathbf{m}^1$ results from the firing of $\omega \in T^*$ starting from the initial marking $\mathbf{m}^0$, we represent it symbolically as $\mathbf{m}^0 \to \omega \to \mathbf{m}^1$. Given an initial marking $\mathbf{m}^0$ the set of *reachable markings* for $\mathbf{m}^0$ denoted by $\Re(N, \mathbf{m}^0)$, is defined as the set of markings generated by all valid firing sequences starting with marking $\mathbf{m}^0$ in the PN N.

The *reachability-tree* of the PN N is a directed-tree $G(N, \mathbf{m}^0)$, where each vertex is associated with an *extended-marking* (i.e. a member of $(\mathcal{N} \cup \infty)^n$), where each place has a non-negative, or, an infinitely large token-load. The vertex, v_0, that is root of the reachability-tree is assigned the initial marking $\mathbf{m}^0$. The algorithm for the construction of the reachability-tree of N can be found in pages 94–95 of Peterson's book [11]. We refrain from presenting it here in the interest of space. Every PN has a finite reachability-tree (Theorem 4-1, [11]). The extended-marking that is associated with a leaf of the reachability-tree is either a deadlocked-marking where their set of enabled transitions is empty, or, it is a duplicate of the extended-marking assigned to some other vertex in the reachability-tree. We can regard vertices in the reachability-tree that are assigned the same extended-marking as duplicates of each other.

The *coverability-graph* of the PN N is the reachability-tree where these duplicate vertices are merges as one. A path $\sigma \in T^*$ in the coverability graph that starts from the root v_0 and terminates at some vertex v_i is denoted as $v_0 \to \sigma \to v_i$. It can be shown that there is a path in the coverability graph that corresponds to every valid firing string in N that starts with the initial marking $\mathbf{m}^0$. However, the converse of this statement is not always true.

A PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is a *free-choice* PN (FCPN) if

$$\forall p \in \Pi, card(p^\bullet) > 1 \Rightarrow^\bullet (p^\bullet) = \{p\}.$$

In other words, a PN is Free-Choice if and only if an arc from a place to a transition is either the unique output arc from that place, or, is the unique input arc to the transition. *Commoner's Liveness Theorem* (cf. [2], chapter 4, [12]) states an FCPN N is live if and only if every minimal siphon in N contains a minimal trap that has a non-empty token load. That is, an FCPN N is live if and only if every minimal siphon contains a marked trap at the initial marking.

We assume a subset of transitions, called *controllable transitions*, $T_c \subseteq T$ can be prevented from firing by an external agent called the supervisor. The set of *uncontrollable transitions*, denoted by $T_u \subseteq T$, is given by $T_u = T - T_c$. If $T_c = T$, then we say we have a *fully-controlled* PN, otherwise we have a *partially controlled* PN. An FCPN is said to be *choice-controlled* if $\forall t \in T_u, ({}^\bullet t)^\bullet = \{t\}$.

A *supervisory policy* $\mathcal{P} : \mathcal{N}^n \to \{0, 1\}^m$, is a total map that returns an m-dimensional binary vector for each reachable marking. The supervisory policy $\mathcal{P}$ permits the firing of transition t_i at marking $\mathbf{m}$, only if $\mathcal{P}(\mathbf{m})_i = 1$. If at a marking $\mathbf{m}$ all input places to a transition t_i contain a token, we say the transition t_i is *state-enabled* at $\mathbf{m}$. If $\mathcal{P}(\mathbf{m})_i = 1$, we say the transition t_i is *control-enabled* at $\mathbf{m}$. A transition has to be state-enabled and control-enabled before it can fire. The fact that uncontrollable transitions cannot be prevented from firing by the supervisory policy is captured by the requirement that $\forall \mathbf{m} \in \mathcal{N}^n, \mathcal{P}(\mathbf{m})_i = 1$, if $t_i \in T_u$. A string of transitions $\sigma = t_{j_1} t_{j_2} \dots t_{j_k}$, where $t_{j_i} \in T$ $(i \in \{1, 2, \dots, k\})$ is said to be a *valid firing string* starting from the marking $\mathbf{m}$, if,

(1) the transition t_{j_1} is enabled at the marking $\mathbf{m}$, $\mathcal{P}(\mathbf{m})_{j_1} = 1$, and
(2) for $i \in \{1, 2, \dots, k-1\}$ the firing of the transition t_{j_i} produces a marking $\tilde{\mathbf{m}}$ at which the transition $t_{j_{i+1}}$ is enabled and $\mathcal{P}(\tilde{\mathbf{m}})_{j_{i+1}} = 1$.

The set of reachable markings under the supervision of $\mathcal{P}$ in N from the initial marking $\mathbf{m}^0$ is denoted by $\Re(N, \mathbf{m}^0, \mathcal{P})$. A transition t_{j_i} is *live* under the supervision of $\mathcal{P}$ if

$$\forall \mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P}), \exists \tilde{\mathbf{m}} \in \Re(N, \mathbf{m}, \mathcal{P}) \text{ such that } t_{j_i} \in T_e(\tilde{\mathbf{m}}) \& \mathcal{P}(\tilde{\mathbf{m}})_{j_i} = 1.$$

A supervisory policy $\mathcal{P}$ enforces liveness if all transitions in N are live under $\mathcal{P}$. In this paper we consider the existence and synthesis of supervisory policies that enforce liveness in partially controlled FCPNs.

3. Review of previous results

Reference [14] contains a necessary and sufficient condition for the existence of a supervisory policy that enforces liveness in an arbitrary PN. This condition is formally stated below.

Theorem 1 (Theorem 5.1, [14]). *Given an arbitrary PN* $N = (\Pi, T, \Phi, \mathbf{m}^0)$, *there is a supervisory policy that enforces liveness if and only if there is a control invariant subset of markings* $\mathcal{M} \subseteq \mathcal{R}(N, \mathbf{m}^0)$ *such that*:

(1) $\forall \mathbf{m}^1 \in \mathcal{M}, \exists \mathbf{m}^2, \mathbf{m}^3 \in \mathcal{M}$, *and* $\exists \omega_1, \omega_2 \in T^*$, *such that* $\mathbf{m}^1 \to \omega_1 \to \mathbf{m}^2 \to \omega_2 \to \mathbf{m}^3$, *and*:
 (a) $\mathbf{m}^3 \geq \mathbf{m}^2$,
 (b) *all transitions in T appear at least once in* ω_2, *and*
 (c) $\forall \omega_3 \in \mathrm{pr}(\omega_1 \omega_2), \mathbf{m}^1 \to \omega_3 \to \mathbf{m}^4 \Rightarrow \mathbf{m}^4 \in \mathcal{M}$, *where* $\mathrm{pr}(\bullet)$ *is the prefix-set of the string argument*.
(2) $\mathbf{m}^0 \in \mathcal{M}$.

This condition is undecidable, but if all the transitions in a PN are controllable, or, if the PN is bounded the above mentioned test is decidable. Furthermore, it is possible to synthesize a least-restrictive policy that enforces liveness for these special cases. For instance, if all transitions in the PN are controllable, testing the condition in Theorem 1 is equivalent to testing the existence of a path $v_0 \to \hat{\sigma}_1 \to v_1 \to \hat{\sigma}_2 \to v_1$ in the coverability graph of N, where all

$Compute\_Control(N = (\Pi, T, \Phi, \mathbf{m}))$

for $t \in T$ **do**
  Suppose $\mathbf{m} \to t \to \widehat{\mathbf{m}}$.
  if $(t \in T_e(N, \mathbf{m})) \vee (test\_condition(N = (\Pi, T, \Phi, \mathbf{m}^0)) == TRUE)$ **then**
    Permit the firing of t.
  else
    Prevent the firing of t.
  end if
end for

Fig. 1. A minimally restrictive supervisory policy that enforces liveness in a PN where all transitions are controllable.

transitions in N appear at least once in $\hat{\sigma}_2$, and $\mathbf{Cx}(\hat{\sigma}_2) \geq \mathbf{0}$, where $\mathbf{C}$ is the *incidence matrix* of the PN [11], and $\mathbf{x}(\bullet)$ represents the *parikh-mapping* (cf. page 127, [7]) of the string-argument (cf. Theorem 5.2, [14]).

Let us suppose $test\_condition(N = (\Pi, T, \Phi, \mathbf{m}^0))$ is a procedure that returns $TRUE$ if such a path exists in the coverability graph of N (cf. Fig. 9, [14]). Using this procedure, Fig. 1 presents a minimally restrictive supervisory policy that enforces liveness in a PN where all transitions are controllable. It is important to highlight the fact that this policy involves the computation of the coverability graph each time the procedure $test\_condition(N = (\Pi, T, \Phi, \mathbf{m}^0))$ is called. This can be computationally intensive as the coverability graph could have a vertex-set whose size is exponentially related to the number of places and transitions in a PN. In précis, the computational cost of testing the existence, and synthesizing the least restrictive policy that enforces liveness can be prohibitive in the general case.

The following theorem from [13] characterizes supervisory policies that enforce liveness in fully controlled FCPNs, which can be viewed as an adaptation of *Commoner's Liveness Theorem* in the context of supervisory control of FCPNs for liveness.

Theorem 2 ([13]). *A supervisory policy* $\mathcal{P} : \mathcal{N}^n \to \{0, 1\}^m$, *enforces liveness in an fully controlled FCPN* $N = (\Pi, T, \Phi, \mathbf{m}^0)$ *if and only if the following conditions are satisfied*:

(1) *If the supervisory policy* $\mathcal{P}$ *prevents the firing of a state-enabled transition* $t \in T$ *at some marking* $\mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P})$, *then* $\exists \hat{m} \in \Re(N, \hat{\mathbf{m}}^0, \mathcal{P})$, *such that the transition t is both state- and control-enabled at the marking* $\hat{\mathbf{m}}$.
(2) $\forall \mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P}), \forall P \subseteq \Pi$, *such that* ${}^\bullet P \subseteq P^\bullet$, $\sum_{p \in P} \mathbf{m}(p) \neq 0$.

The proof of the above result, *mutatis mutandis*, holds for partially controlled FCPNs. Therefore, the above characterization is valid for supervisory policies that enforce liveness in partially controlled FCPNs as well.

Theorem 4.1 in reference [16] states that if there is a supervisory policy that enforces liveness in a partially controlled FCPN $N = (\Pi, T, \Phi, \mathbf{m}^0)$, then there is a supervisory policy that enforces liveness in the FCPN $\hat{N} = (\Pi, T, \Phi, \hat{\mathbf{m}}^0)$, where $\hat{\mathbf{m}}^0 \geq \mathbf{m}^0$. This means the set of initial markings for which there is a policy that enforces liveness is right-closed. Every right-closed set can be represented by a finite-set of minimal elements, and reference [15] presents a minimally restrictive supervisory policy that enforces liveness in a partially controlled FCPN, provided the minimal elements of the above mentioned right-closed set are readily available, and the initial marking is greater than or equal to some minimal element. The issue of deciding if the right-closed set is non-empty for an arbitrary, partially controlled FCPN is still open to our knowledge. That is, given an arbitrary partially controllable FCPN, we cannot tell if there is a supervisory policy that enforces liveness. While the results in the next section do not present a comprehensive solution to this problem, they present a decision procedure that can determine the existence of a supervisor for the class of choice-controlled FCPNs.

Reference [5] concerns the issue of enforcing liveness in a subset of transitions using supervisory control that uses the linear-algebraic approach in reference [8]. The procedure presented in this reference is not guaranteed to halt in all instances, but when it does halt, it presents a supervisory policy that enforces liveness in the original PN. In the event the procedure halts, a posteriori, one could say that the procedure for liveness enforcement in this reference identifies a class of PNs for which there is a supervisory policy that enforces liveness. Reference [10] identifies the class of S^2PGR^2-nets as a family of PNs that can be made live via under supervision under appropriate conditions. References [3,17,4] discuss various aspects of monitor-based supervisory control procedure that enforces liveness in bounded PNs using the process of unfolding. In the sprit of theses references, the present paper also identifies two families of PNs that can be made live via supervision.

4. Main results

Our first observation is that minimally restrictive supervisory policies that enforce liveness in a partially controlled FCPN do not disable certain transitions.

Observation 3. *If $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is a partially controlled FCPN, and $\mathcal{P} : \mathcal{N}^n \to \{0, 1\}^m$ is a minimally restrictive supervisory policy that enforces liveness in N, then the supervisory policy $\mathcal{P}$ does not control-disable transitions $t \in T$ such that $({}^\bullet t)^\bullet = \{t\}$.*

Proof. Let $\mathbf{m}^1 \in \mathcal{R}(N, \mathbf{m}^0, \mathcal{P})$, and let $t \in T$ be a transition that is state-enabled under the marking $\mathbf{m}^1$ (i.e. $t \in T_e(N, \mathbf{m}^1)$) such that $({}^\bullet t)^\bullet = \{t\}$.

Let us suppose $\mathbf{m}^1 \to t \to \mathbf{m}^2$ in the uncontrolled FCPN N. Since $\mathcal{P}$ enforces liveness in N, $\exists \omega_1 \in T^*$, such that t does not appear in ω_1, and

$$\mathbf{m}^1 \to \omega_1 \to \mathbf{m}^3 \to t \to \mathbf{m}^4$$

in N under the supervision of $\mathcal{P}$. Since $({}^\bullet t)^\bullet = \{t\}$ it follows that $\mathbf{m}^2 \to \omega_1 \to \mathbf{m}^4$ in N.

Suppose $\exists \omega_2 \in \mathrm{pr}(\omega_1)$, and $\exists t_u \in T_u$, where $\mathbf{m}^2 \to \omega_2 \to \mathbf{m}^5 \to t_u \to \mathbf{m}^6$ in N, where pr($\bullet$) denotes the prefix-set of the string argument. Further, let us suppose $\mathbf{m}^1 \to \omega_2 \to \mathbf{m}^7 \to t_u \to \mathbf{m}^8$ as shown in Fig. 2(a), then $\mathbf{m}^1 \to \omega_2 \to \mathbf{m}^7 \to t_u \to \mathbf{m}^8$ will be permitted by $\mathcal{P}$. Since $\mathcal{P}$ enforces liveness, $\exists \omega_3 \in T^*$ such that t does not appear in ω_3, and $\mathbf{m}^8 \to \omega_3 \to \mathbf{m}^9 \to t \to \mathbf{m}^{10}$. Since $({}^\bullet t)^\bullet = \{t\}$, it follows that $\mathbf{m}^6 \to \omega_3 \to \mathbf{m}^{10}$ (cf. Fig. 2(a)).

If $t_u \notin T_e(N, \mathbf{m}^7)$, then $\exists p \in \Pi$ such that $\{(t, p), (p, t_u)\} \subseteq \Phi$. Since N is an FCPN it follows that $\mathbf{m}^4 \to t_u \to \mathbf{m}^{11}$ under the supervision of $\mathcal{P}$ (cf. Fig. 2(b)). But this would mean that $\mathbf{m}^6 \to \omega_4 \to \mathbf{m}^{11}$, where $\omega_1 = \omega_2\omega_4$.

Repeating the above argument for all potential candidates for $t_u \in T_u$, we conclude that $\mathbf{m}^2 \in \mathcal{M}$ as per the requirements of Theorem 5.1 of reference [14]. Since $\mathcal{P}$ is minimally restrictive, it follows that it should permit the firing of t at the marking $\mathbf{m}^2$. $\square$

The above observation shows that a minimally restrictive supervisory policy that enforces liveness in a partially controlled FCPN only prevents the firing of *choice-transitions*. That is, transitions $t \in T$ that satisfy the condition $\{t\} \subset ({}^\bullet t)^\bullet$. This observation also holds for completely controlled FCPNs. That is, if there is a minimally restrictive supervisory policy that enforces liveness in a completely controlled FCPN, it would only disable choice-transitions.

Also, for an arbitrary PN, there is a minimally restrictive supervisory policy that enforces liveness if and only if there exists some supervisory policy that enforces liveness (cf. Theorem 6.1, [14]). This naturally lends itself to a test for the existence, and synthesis, of a minimally restrictive supervisory policy that enforces liveness in a choice-controlled FCPN.

Given a choice-controlled FCPN, we can convert it into a completely controllable equivalent by assuming all transitions to be controllable. There is a supervisory policy that enforces liveness in the choice-controlled FCPN if and

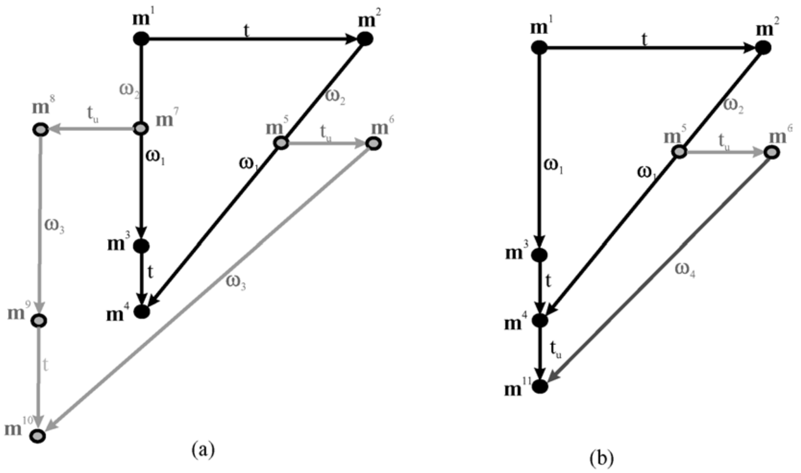

Fig. 2. Two instances of a partial-map of the reachable markings used in the proof of Observation 3.

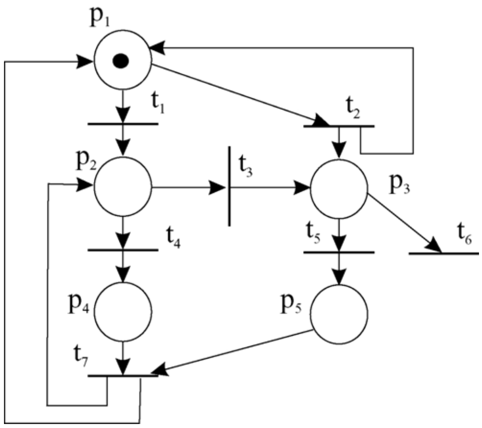

Fig. 3. An FCPN example.

only if there is a similar policy for the completely controllable equivalent. Since the problem of deciding the existence of a supervisor that enforces liveness in completely controllable PNs is decidable (cf. Corollary 5.1, [14]), it follows that we can decide if there is a liveness enforcing supervisory policy for choice-controlled FCPNs. Fig. 10 of reference [14] contains a minimally restrictive supervisory policy for any completely controllable PN. This policy when used on the choice-controlled FCPN will be a minimally restrictive policy. This is formally stated as follows.

Theorem 4. *A (minimally restrictive) supervisory policy that enforces liveness in the completely controlled equivalent is also a (minimally restrictive) policy that enforces liveness in the original choice-controlled FCPN.*

Testing the existence of a supervisory policy for an arbitrary FCPN is still open to our knowledge. Observation 3 might seem to suggest that if there is a minimally restrictive supervisory policy the enforces liveness in a partially controlled FCPN, then the transitions that are control-disabled are exit transitions of siphons. That is, transitions in the set $P^\bullet -^\bullet P$, for any $P \subseteq \Pi$ where ${}^\bullet P \subseteq P^\bullet$.

Any supervisory policy that enforces liveness in the FCPN shown in Fig. 3 must disable transition t_1 at the initial marking. But, transition t_1 is not an exit transition of the two siphons: $P_1 = \{p_1, p_2, p_4\}$ and $P_2 = \{p_1, p_2, p_3, p_5\}$. None of these siphons contain traps (there are no traps in the FCPN). The exit transition-set of P_1 is $\{t_3\}$ and that of P_2 is $\{t_4, t_6\}$. As long as t_1 is uncontrollable, there can be no supervisory policy that enforces liveness. This example also puts to rest a conjecture that if all exit transitions of siphons that do not contain traps are controllable, then there is a supervisory policy that enforces liveness.

4.1. Non-choice-controlled FCPNs

We now turn our attention to a class of FCPNs that are not necessarily choice-controlled, that can be made live via supervision. This family of FCPNs are a generalization of the class of *Independent, Increasing FCPNs* (II-FCPNs) introduced in [13]. To this end, we require the procedure *Compute_Subnet*(N, Π_1, T_1) shown in Fig. 4.

Since the number of places and transitions in the FCPN N is finite, the while-loop in the procedure *Compute_Subnet*(N, Π_1, T_1) is guaranteed to terminate. The exit-condition for the while-loop in the procedure that computes $\hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{\mathbf{m}}^0)$ ensures $({}^\bullet\hat{T})_N = \hat{\Pi}$.

Let N be an FCPN, and $P \subseteq \Pi$ be a siphon (i.e. ${}^\bullet P \subseteq P^\bullet$), if $\hat{N} = \textit{Compute\_Subnet}(N, P, {}^\bullet P)$, where $\hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{\mathbf{m}}^0)$, it can be shown that $\hat{\Pi}$ is also a siphon (i.e. ${}^\bullet\hat{\Pi} \subseteq \hat{\Pi}^\bullet$) in N. We say a siphon $P \subseteq \Pi$ (${}^\bullet P \subseteq P^\bullet$) in an FCPN N has property $\mathcal{A}$ if the following statements hold for $\hat{N} = \textit{Compute\_Subnet}(N, P, {}^\bullet P)$, where $\hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{\mathbf{m}}^0)$

(1) $((\hat{\Pi}^\bullet)_N - ({}^\bullet\hat{\Pi})_N) \subseteq T_c$,
(2) $\hat{N}$ is a live FCPN, and
(3) $\forall \hat{p} \in^\bullet ((\hat{\Pi}^\bullet)_{\hat{N}} - ({}^\bullet\hat{\Pi})_{\hat{N}})_{\hat{N}}$, $\hat{p}$ must be unbounded in $\hat{N}$.

$Compute\_Subnet(N = (\Pi, T, \Phi, \mathbf{m}^0), \Pi_1 \subseteq \Pi, T_1 \subseteq T)$

Create $\widehat{\Pi} = \Pi_1$
Create $\widehat{T} = T_1$
while $({}^\bullet\widehat{T})_N \not\subseteq \widehat{\Pi}$ **do**
  $\widehat{\Pi} \leftarrow \widehat{\Pi} \cup ({}^\bullet\widehat{T})_N$
  $\widehat{T} \leftarrow \widehat{T} \cup ({}^\bullet\widehat{\Pi})_N$
end while
Create $\widehat{\Phi} = \{(\widehat{\Pi} \times \widehat{T}) \cup (\widehat{T} \times \widehat{\Pi})\} \cap \Phi$
Define $\widehat{\mathbf{m}}^0 : \widehat{\Pi} \to \mathcal{N}$ as follows: $\forall p \in \widehat{\Pi}, \widehat{\mathbf{m}}^0(p) = \mathbf{m}^0(p)$.
return PN $\widehat{N} = (\widehat{\Pi}, \widehat{T}, \widehat{\Phi}, \widehat{\mathbf{m}}^0)$

Fig. 4. The procedure $\hat{N} = Compute\_Subnet(N, \Pi_1, T_1)$, where $\hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{\mathbf{m}}^0)$.

The first condition of property $\mathcal{A}$ requires the exit-transitions of the siphon $\hat{\Pi}$ to be controllable. From the second condition, we note that we can control-disable all transitions in the set $((\hat{\Pi}^\bullet)_N - ({}^\bullet\hat{\Pi})_N)(\subseteq T_c)$ in the original PN N can obtain a behavior that parallels the (live) behavior of $\hat{N}$. The third condition guarantees that the places in the set ${}^\bullet((\hat{\Pi}^\bullet)_{\hat{N}} - ({}^\bullet\hat{\Pi})_{\hat{N}})_{\hat{N}}$ will be unbounded under this behavior. That is, the token loads of the places in the set ${}^\bullet((\hat{\Pi}^\bullet)_{\hat{N}} - ({}^\bullet\hat{\Pi})_{\hat{N}})_{\hat{N}}$ can be made as large as desired under the aforementioned supervisory action. We say a partially controlled FCPN N has property $\mathcal{A}$ if every minimal siphon of N either (i) contains a marked trap at the initial marking, or (ii) is a siphon with property $\mathcal{A}$.

As an illustration consider the partially controlled FCPN shown in Fig. 5. The uncontrollable (controllable) transitions are shown as unfilled (filled) rectangles. This FCPN has three minimal siphons $P_1 = \{p_5, p_6\}$, $P_2 = \{p_1, p_3, p_4, p_7\}$ and $P_3 = \{p_2, p_3, p_4, p_5, p_7, p_8, p_9\}$. The siphon P_1 is also a trap and it is marked. The siphons P_2 and P_3 do not contain a trap, and therefore this FCPN is not live in the absence of supervision.

Let us suppose $\hat{N} = Compute\_Subnet(N, P_2, {}^\bullet P_2)$, where $\hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{\mathbf{m}}^0)$. We note that $\hat{N}$ is essentially the FCPN shown in Fig. 5 without transition t_6. This FCPN is live as it satisfies Commoner's Liveness Theorem. Note that $t_6 = ((\hat{\Pi}^\bullet)_{\hat{N}} - ({}^\bullet\hat{\Pi})_{\hat{N}})_{\hat{N}} \subseteq T_c$, and $\{p_7\} = {}^\bullet((\hat{\Pi}^\bullet)_{\hat{N}} - ({}^\bullet\hat{\Pi})_{\hat{N}})_{\hat{N}}$ is unbounded in $\hat{N}$. We note that $\hat{N}$ is also equal to $Compute\_Subnet(N, P_3, {}^\bullet P_3)$ for this example. So, the FCPN shown in Fig. 5 has property $\mathcal{A}$.

The FCPN shown in Fig. 3 does not satisfy property $\mathcal{A}$ as $\hat{N} = Compute\_Subnet(N, P_1, {}^\bullet P_1)$ where the set $P_1 = \{p_1, p_2, p_4\}$ is a siphon, results in an FCPN that is essentially the FCPN shown in Fig. 3 without transition t_6. This FCPN is not live.

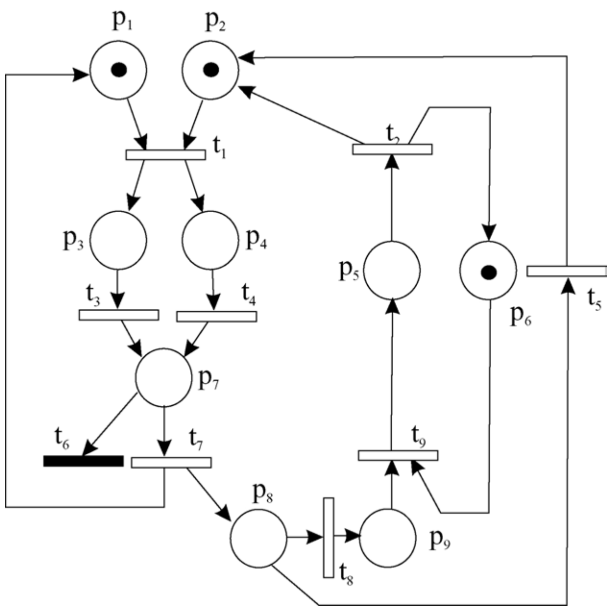

Fig. 5. A FCPN with property $\mathcal{A}$.

The FCPN in Fig. 5 does not belong to the family of II-FCPNs identified in reference [13]. This FCPN is not choice-controlled either as the transitions in the set $p_8^\bullet = \{t_8, t_5\}$ are not controllable. For this example, it can be shown that the supervisory policy that permits the firing of t_6 only when the sum of the tokens in the place-sets $P_2 = \{p_1, p_3, p_4, p_7\}$ exceeds unity (which also implies that the sum of the tokens in $P_3 = \{p_2, p_3, p_4, p_5, p_7, p_8, p_9\}$ exceeds unity), enforces liveness. The following result is a generalization of this observation to FCPNs that satisfy property $\mathcal{A}$.

Theorem 5. *A partially controlled FCPN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ with property $\mathcal{A}$ can be made live via supervision.*

Proof. For each minimal siphon $P_i \subseteq \Pi$ of N that does not contain a trap we compute $\hat{N}_i = Compute\_Subnet(N, P_i, {}^\bullet P_i)$, where $\hat{N}_i = (\hat{\Pi}_i, \hat{T}_i, \hat{\Phi}_i, \hat{\mathbf{m}}^0)$. We permit the firing of $t \in ((\hat{\Pi}_i^\bullet)_N - ({}^\bullet\hat{\Pi}_i)_N)(\subseteq T_c)$ only if

(1) the projection of the marking resulting from the firing of t in N onto places in $\hat{\Pi}_i$ keeps the subnet $\hat{N}_i$ live, and
(2) the place $p = ({}^\bullet t)_N \cap \hat{\Pi}_i$ remains unbounded in the subnet $\hat{N}_i$ at this new marking.

Any transition that does not fit the description of t as listed above, is permanently control-enabled under the policy. Using Theorem 2, which also holds for partially controlled FCPNs, we show that this supervisory policy enforces liveness.

The first requirement in the supervisory policy guarantees the fact that no minimal siphon is every emptied. This observation can be established by the contrapositive argument. If a minimal siphon $P_i \subseteq \Pi$ was emptied at some marking $\mathbf{m}^i$, it must be that P_i did not contain a trap. Additionally, the subnet of N that is identical to $\hat{N}_i = Compute\_Subnet(N, P_i, {}^\bullet P_i)$ would not be live at marking $\mathbf{m}^i$ and the supervisory policy would not permit the firing of the transition that resulted in the marking $\mathbf{m}^i$.

If a transition $t \in ((\hat{\Pi}_i^\bullet)_N - ({}^\bullet\hat{\Pi}_i)_N)(\subseteq T_c)$ is prevented from firing at a marking $\mathbf{m}^i$ by the supervisory policy, then one of the two conditions in the statement of the policy must be violated under the new marking $\mathbf{m}^{i+1}$ arising from the firing of the transition t (i.e. $\mathbf{m}^i \to t \to \mathbf{m}^{i+1}$). There is always a marking $\mathbf{m}^j$, reachable from the marking $\mathbf{m}^i$, at which the transition t is both control- and state-enabled. This is because by control-disabling all transitions in the set $((\hat{\Pi}_i^\bullet)_N - ({}^\bullet\hat{\Pi}_i)_N)(\subseteq T_c)$ in N we can realize the live sub-behavior of $\hat{N}_i = Compute\_Subnet(N, P_i, {}^\bullet P_i)$, where $\hat{N}_i = (\hat{\Pi}_i, \hat{T}_i, \hat{\Phi}_i, \hat{\mathbf{m}}_i^0)$. Since $p = ({}^\bullet t)_N \cap \hat{\Pi}_i$ is unbounded in $\hat{N}_i$ under this sub-behavior, there is a marking reachable from the current marking where the token load of p can be made larger than any pre-determined value. There can be no policy-induced deadlocks as a consequence. That is, there cannot be a situation where all (controllable) output transitions of a place are simultaneously control-disabled.

So, there is a reachable marking where every control-disabled transition will be state- and control-enabled. From Theorem 2, we conclude that the supervisory policy enforces liveness. □

The converse of Theorem 5 is not true. The FCPN shown in Fig. 3 does not satisfy property $\mathcal{A}$, but there is a supervisory policy that enforces liveness as long as transitions t_1 and t_3 are controllable.

We now turn our attention to the computational effort involved in determining if a given FCPN has property $\mathcal{A}$. Checking liveness of an arbitrary FCPN is NP-complete (cf. [1], Section A12, problem MS3), and testing boundedness of an arbitrary PN is PSPACE-complete [6]. Additionally, computing the set of minimal siphons in an FCPN is intractable in the general setting. All of this does not bode well for an online implementation. However, these tests can be performed off-line and once the various $\hat{N}_i$'s are computed, the supervisory policy outlined in the proof of Theorem 5 can be characterized and enforced in comparatively easy manner.

5. Conclusions

In this paper we showed that if there is a minimally restrictive supervisory policy that enforces liveness in a partially controlled Free-Choice PN, then no transition t that satisfies the property $({}^\bullet t)^\bullet = \{t\}$ is ever prevented from firing under supervision. This observation was used to show that it is possible to decide if there is a supervisory policy that enforces liveness in a restricted class of FCPNs called choice-controlled FCPNs. A choice-controlled FCPN has the property that all transitions that satisfy the requirement $\{t\} \subset ({}^\bullet t)^\bullet$ are controllable. Some counterexamples to conjectures that stem from the above observation for arbitrary, partially controlled FCPNs were also presented. Following this, we identified a new class of partially controlled FCPNs for which there is a pre-determined supervisory policy that enforces liveness that can be enforced easily after it has been computed off-line.

It is hoped that results in this paper brings us a little closer to resolving the problem of deciding the existence of a supervisory policy policy that enforces liveness in an arbitrary, partially controlled FCPN.

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