Proceedings of the 36th
Conference on Decision & Control
San Diego, California USA • December 1997

On a Free-Choice Equivalent of a Petri Net

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August 25, 1997

Abstract

A *Petri net* (PN) [2, 3] is said to be *live* if it is possible to fire any transition from every reachable marking, although not necessarily immediately. In this paper we consider the issue of enforcing *liveness* in non-live PNs via supervisory control. Using a construction procedure, similar to that in reference [1], that converts an arbitrary PN into an equivalent member of a family of PNs called *Free-Choice Petri nets* (cf. section 7.2, [3]), we show there is a policy that enforces liveness in the original PN if and only if there is a corresponding policy for its Free-Choice equivalent. The utility of this approach is illustrated via an example.

1 Introduction

A large class of systems can be modeled as systems with independent, interacting, concurrent components. Typically, each independent process is split into several operations, the execution of each operation is conditioned on the satisfaction of a set of logical preconditions. Upon the execution of any such operation, a new set of logical conditions is created that inhibit the execution of some operations and enables the execution of others in the system. The supervisory control of such systems requires an external agent to regulate, or limit, the operations of each component so as to guarantee a common objective. In this paper we concern ourselves with a stronger version of deadlock avoidance called *liveness*. From any reachable state of a *live* system, it should be possible for any of the components to execute any of its operations, although not necessarily immediately.

Petri nets (PNs) [2, 3] are an ideal choice of the modeling of such systems as they allow easy representation of the logical preconditions as the marking, and the operations can be represented as transitions. A PN is said to be *live*, if from any reachable marking it is possible to fire any transition, although not necessarily immediately. We assume every transition in the PN can be prevented from firing by the supervisor, and we concern ourselves with the synthesis of supervisory policies that enforce liveness in non-live PNs. A necessary and sufficient condition for the existence of such a policy for an arbitrary PN can be found in references [6, 4]. This condition can be used to synthesize a minimally-restrictive policy that enforces liveness, but this synthesis can be computationally burdensome. This is primarily due to the fact that at each new marking this policy requires the computation of the *coverability graph* (cf. section 5.3, [3]) of the non-live PN.

Reference [5] presents a characterization of policies that enforce liveness in a class of PNs called *Free-Choice Petri nets* (FCPNs). A *Free-Choice Petri net* (FCPN) is a restricted class of Petri nets (PNs) where every arc from a place to a transition is either the unique output arc from that place, or, it is the unique input arc to the transition. Reference [5] also introduces a class of FCPNs for which a supervisory policy that enforces liveness can be readily constructed. There can be many other classes of FCPNs that can be made live via supervision that are yet to be discovered. We envision a catalogue of such families of FCPNs for which there exists a pre-constructed supervisory policy that enforces liveness. The results of this paper provide the first-step in developing a prescrip-

*This work was supported in part by grant RES-BRD-IC-SREENIVAS-1-2-68317 from the UIUC Campus Research Board and the National Science Foundation under grant number ECS-9409691.

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tion for the conversion of these FCPN-specific policies to PNs that are not FCPNs.

2 Notations and Definitions

A *Petri net* (PN) $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is an ordered 4-tuple, where $\Pi = \{p_1, p_2, \ldots, p_n\}$ is a set of n *places*, $T = \{t_1, t_2, \ldots, t_m\}$ is a set of m *transitions*, $\Phi \subseteq (\Pi \times T) \cup (T \times \Pi)$ is a set of *arcs*, $\mathbf{m}^0{:}\Pi \rightarrow \mathcal{N}$ is the *initial marking function* (or the *initial marking*), and $\mathcal{N}$ is the set of nonnegative integers. The *state* of a PN is the marking $\mathbf{m}{:}\Pi \rightarrow \mathcal{N}$ that identifies the number of *tokens* in each place. A marking $\mathbf{m}{:}\Pi \rightarrow \mathcal{N}$ is sometimes represented by an integer-valued vector $\mathbf{m} \in \mathcal{N}^n$, where the i-th component $\mathbf{m}_i$ represents the token load $(\mathbf{m}(p_i))$ of the i-th place. The context should suggest the appropriate usage. For a given marking $\mathbf{m}$ a transition $t \in T$ is said to be *enabled* if $\forall\, p \in {}^\bullet t$, $\mathbf{m}(p) \geq 1$, where ${}^\bullet x = \{y \mid (y, x) \in \Phi\}$. The set of enabled transitions is denoted by the symbol $T_e(\mathbf{m})$. An enabled transition $t \in T_e(\mathbf{m}^1)$ can *fire*, which changes the marking $\mathbf{m}^1$ to $\mathbf{m}^2$ according to the equation

$$\mathbf{m}^2(p) = \mathbf{m}^1(p) - card(p^\bullet \cap \{t\}) + card({}^\bullet p \cap \{t\}), \quad (1)$$

where $x^\bullet = \{y \mid (x, y) \in \Phi\}$, and the symbol $card(\bullet)$ is used to denote the cardinality of the set argument. This notation is also used to denote the predecessor or successor set of a set of places or transitions.

A string of transitions $\sigma = t_{j_1} t_{j_2} \cdots t_{j_k}$, where $t_{j_i} \in T$ $(i \in \{1, 2, \ldots, k\})$ is said to be a *valid firing string* starting from the marking $\mathbf{m}$, if,

- the transition t_{j_1} is enabled at the marking $\mathbf{m}$, and
- for $i \in \{1, 2, \ldots, k-1\}$ the firing of the transition t_{j_i} produces a marking at which the transition $t_{j_{i+1}}$ is enabled.

The set of *reachable markings* from $\mathbf{m}^0$, denoted by $\Re(N, \mathbf{m}^0)$, is the set of markings generated by all valid firing strings starting with marking $\mathbf{m}^0$ in the PN N. At a marking $\mathbf{m}^1$, if the firing of a valid firing string σ results in a marking $\mathbf{m}^2$, we represent it as $\mathbf{m}^1 \rightarrow \sigma \rightarrow \mathbf{m}^2$. The set of all valid firing strings starting from the marking $\mathbf{m}^1$ in a PN N is denoted by $L(N, \mathbf{m}^1)$, where $L(N, \mathbf{m}^1) = \{\sigma \in T^* \mid \mathbf{m}^1 \rightarrow \sigma \rightarrow \mathbf{m}^2 \text{ for some } \mathbf{m}^2 \in \Re(N, \mathbf{m}^1)\}$. A transition $t \in T$ is *live* if $\forall \mathbf{m}^1 \in \Re(N, \mathbf{m}^0), \exists$ a $\mathbf{m}^2 \in \Re(N, \mathbf{m}^1)$ such that $t \in T_e(\mathbf{m}^2)$. A PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ is a *Free-Choice* PN (FCPN) if $\forall p \in \Pi, card(p^\bullet) > 1 \Rightarrow {}^\bullet(p^\bullet) = \{p\}$.

In other words, a PN is Free-Choice if and only if an arc from a place to a transition is either the unique output arc from that place, or, is the unique input arc to the transition.

A *supervisory policy* $\mathcal{P}{:}\mathcal{N}^n \rightarrow \{0,1\}^m$, is a total map that returns an m-dimensional binary vector for each reachable marking. The supervisory policy $\mathcal{P}$ permits the firing of transition t_i at marking $\mathbf{m}$, only if $\mathcal{P}(\mathbf{m})_i = 1$. If at a marking $\mathbf{m}$ all input places to a transition t_i contain a token, we say the transition t_i is *state-enabled* at $\mathbf{m}$. If $\mathcal{P}(\mathbf{m})_i = 1$, we say the transition t_i is *control-enabled* at $\mathbf{m}$. A transition has to be state-enabled and control-enabled before it can fire. A string of transitions $\sigma = t_{j_1} t_{j_2} \cdots t_{j_k}$, where $t_{j_i} \in T$ $(i \in \{1, 2, \ldots, k\})$ is said to be a *valid firing string* starting from the marking $\mathbf{m}$, if,

- the transition t_{j_1} is enabled at the marking $\mathbf{m}$, $\mathcal{P}(\mathbf{m})_{j_1} = 1$, and
- for $i \in \{1, 2, \ldots, k-1\}$ the firing of the transition t_{j_i} produces a marking $\mathbf{m}^i$ at which the transition $t_{j_{i+1}}$ is enabled and $\mathcal{P}(\mathbf{m}^i)_{j_{i+1}} = 1$.

The set of reachable markings under the supervision of $\mathcal{P}$ in N from the initial marking $\mathbf{m}^0$ is denoted by $\Re(N, \mathbf{m}^0, \mathcal{P})$. A transition t_{j_i} is *live* under the supervision of $\mathcal{P}$ if $\forall \mathbf{m}^1 \in \Re(N, \mathbf{m}^0, \mathcal{P}), \exists \mathbf{m}^2 \in \Re(N, \mathbf{m}^1, \mathcal{P})$ such that $t_{j_i} \in T_e(\mathbf{m}^2)$ and $\mathcal{P}(\mathbf{m}^2)_{j_i} = 1$. A supervisory policy $\mathcal{P}$ enforces liveness if all transitions in N are live under $\mathcal{P}$. The following theorem from reference [5] characterizes supervisory policies that enforce liveness in FCPNs.

Theorem 2.1 *[5] A supervisory policy $\mathcal{P}{:}\mathcal{N}^n \rightarrow \{0,1\}^m$, enforces liveness in an FCPN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ if and only if the following conditions are satisfied:*

1. $\forall\, \mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P}), \forall\, t_i \in T, \mathcal{M}(\mathbf{m}, t_i) \neq \emptyset \Rightarrow \exists\, \widehat{\mathbf{m}} \in \mathcal{M}(\mathbf{m}, t_i)$, *such that* $\mathcal{P}(\widehat{\mathbf{m}})_i = 1$, *and*
2. $\forall\, \mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P}), \forall\, P \subseteq \Pi$, *such that* ${}^\bullet P \subseteq P^\bullet$, $\sum_{p \in P} \mathbf{m}(p) \neq 0$,

where $\forall\, \mathbf{m} \in \Re(N, \mathbf{m}^0, \mathcal{P}), \forall\, t_i \in T, \mathcal{M}(\mathbf{m}, t_i) = \{\mathbf{m}^1 \in \Re(N, \mathbf{m}, \mathcal{P}) \mid {}^\bullet t_i \cap \Delta(\mathbf{m}^1) = \emptyset\}$, *and* $\Delta(\mathbf{m}^1) = \{p \in \Pi \mid \mathbf{m}^1(p) = 0\}$, *is the collection of places that are empty at the marking* $\mathbf{m}^1$.

The following theorem from references [6, 4] characterizes supervisory policies that enforce liveness in an arbitrary PN where every transition can be prevented from firing by the supervisor.

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Theorem 2.2 *[6, 4] For a given PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$, there exists a supervisory policy $\mathcal{P}: \mathcal{N}^n \rightarrow \{0,1\}^m$ that enforces liveness, if and only if $\exists$ a valid firing string $\sigma = \sigma_1\sigma_2$, in N, starting from $\mathbf{m}^0$, such that (i) $\mathbf{m}^0 \rightarrow \sigma_1 \rightarrow \mathbf{m}^1 \rightarrow \sigma_2 \rightarrow \mathbf{m}^2$, (ii) $\mathbf{m}^2 \geq \mathbf{m}^1$, and (iii) all transitions appear at least once in the firing string σ_2.*

3 Main Results

Let $N = (\Pi, T, \Phi, \mathbf{m}^0)$ be a PN that is not necessarily Free-Choice. Using a procedure that is similar to the one used in appendix I of reference [1], we convert the PN N into an FCPN $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ by adding extra places and transitions and making alterations to the arcs as defined in the procedure convert_to_FCPN (N) (cf. figure 1). Observation 3.1 establishes the fact that the PN $\widetilde{N}$ obtained from above procedure is an FCPN. The proof follows directly from the construction and is skipped for brevity. Figure 2(a) contains a PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$ that is not an FCPN. To see this note that ${}^\bullet(p_9^\bullet) = {}^\bullet(p_{10}^\bullet) = \{p_9, p_{10}\}$. Letting $(\widetilde{N}, \widehat{T}, \widehat{\Pi}) =$ convert_to_FCPN (N), we get the PN $\widetilde{N}$ shown in figure 2(b), where $\widehat{T} = \{\widehat{t}_1, \widehat{t}_2\}$ and $\widehat{\Pi} = \{\widehat{p}_1, \widehat{p}_2\}$.

Observation 3.1 *If $(\widetilde{N}, \widehat{T}, \widehat{\Pi}) =$ convert_to_FCPN (N), where $N = (\Pi, T, \Phi, \mathbf{m}^0)$ and $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ are two PNs, then $\widetilde{N}$ is Free-Choice.*

Since the PN N and the FCPN $\widetilde{N}$ share common places and transitions it is necessary to define the following sets to eliminate any confusion. The symbol $({}^\bullet p)_N$ denotes the set $\{t \in T \mid (t,p) \in \Phi\}$, and the symbol $({}^\bullet p)_{\widetilde{N}}$ denotes the set $\{t \in \widetilde{T} \mid (t,p) \in \widetilde{\Phi}\}$. The symbols $(p^\bullet)_N$, $(p^\bullet)_{\widetilde{N}}$, $({}^\bullet t)_N$, $({}^\bullet t)_{\widetilde{N}}$, $(t^\bullet)_N$, and $(t^\bullet)_{\widetilde{N}}$ are defined accordingly. If $\widetilde{\sigma} \in \widetilde{T}^*$ is a firing sequence in $\widetilde{N}$, then $\widetilde{\sigma}\mid_T \in T^*$ is the projection of the string $\widetilde{\sigma}$ to the alphabet T. That is, $\widetilde{\sigma}\mid_T$ is the string of transitions in T obtained from $\widetilde{\sigma}$ by erasing all transitions that are not in T while retaining the transition order in the rest of the string. The set of strings $L(\widetilde{N}, \widetilde{\mathbf{m}}^0)\mid_T \subseteq T^*$ is defined accordingly. Let $\mathbf{m} \in \mathcal{N}^{card(\Pi)}$ be a marking of the PN N, we define a corresponding marking $\beta(\mathbf{m})$ of the PN $\widetilde{N}$ as follows

$$\beta(\mathbf{m})(p) = \begin{cases} \mathbf{m}(p) & \text{if } p \in \Pi \\ 0 & \text{otherwise.} \end{cases}$$

For example, for the definition of $\widetilde{\mathbf{m}}^0$, we note $\widetilde{\mathbf{m}}^0 = \beta(\mathbf{m}^0)$. Theorem 3.1 (3.2) relates the existence of a

(FCPN, transition-set, place-set) convert_to_FCPN (N)
PN $N = (\Pi, T, \Phi, \mathbf{m}^0)$; {
Create $\widetilde{\Pi}$, $\widetilde{T}$, $\widetilde{\Phi}$, $\widehat{T}$, $\widehat{\Pi}$ and initialize as $\widetilde{\Pi} \leftarrow \Pi$, $\widetilde{T} \leftarrow T$, $\widetilde{\Phi} \leftarrow \Phi$, $\widehat{T} \leftarrow \emptyset$, $\widehat{\Pi} \leftarrow \emptyset$;
/ $\widetilde{\Pi}$, $\widetilde{T}$ and $\widetilde{\Phi}$ denote the set of places, transitions and arcs of the PN $\widetilde{N}$. $\widehat{T}$ and $\widehat{\Pi}$ denote the set of newly added transitions and places. */*
For each $p \in \Pi$ do {
  if ((card($p^\bullet$) > 1) && (${}^\bullet(p^\bullet) \neq \{p\}$)) {
    $\forall\, t \in p^\bullet$ {
      if (${}^\bullet t \neq \{p\}$) {
        Introduce a new and unused transition $\widehat{t}$ to $\widetilde{N}$ (i.e. $\widehat{t} \notin \widetilde{T}$, and $\widetilde{T} \leftarrow \widetilde{T} \cup \{\widehat{t}\}$);
        Add $\widehat{t}$ to the newly added set of transitions ($\widehat{T} \leftarrow \widehat{T} \cup \{\widehat{t}\}$);
        Introduce a new and unused place $\widehat{p}$ to $\widetilde{N}$ (i.e. $\widehat{p} \notin \widetilde{\Pi}$, and $\widetilde{\Pi} \leftarrow \widetilde{\Pi} \cup \{\widehat{p}\}$);
        Add $\widehat{p}$ to the newly added set of places ($\widehat{\Pi} \leftarrow \widehat{\Pi} \cup \{\widehat{p}\}$);
        $\widetilde{\Phi} \leftarrow \{\widetilde{\Phi}$ - {(p,t)}} $\cup$ {(p, $\widehat{t}$), ($\widehat{t}$, $\widehat{p}$), ($\widehat{p}$, t)};
      }
    }
  }
}
/ Note that $\widetilde{\Pi} = \Pi \cup \widehat{\Pi}$ and $\widetilde{T} = T \cup \widehat{T}$, where $\Pi \cap \widehat{\Pi} = T \cap \widehat{T} = \emptyset$. */*
For each $\widetilde{p} \in \widetilde{\Pi}$ do {
  if ($\widetilde{p} \in \Pi$)
    $\widetilde{\mathbf{m}}^0(\widetilde{p}) = \mathbf{m}^0(\widetilde{p})$;
  else
    $\widetilde{\mathbf{m}}^0(\widetilde{p}) = 0$;
}
return $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$, $\widehat{T}$, and $\widehat{\Pi}$; }

Figure 1: The procedure $(\widetilde{N}, \widehat{T}, \widehat{\Pi}) =$ *convert_to_FCPN* (N), returns an equivalent FCPN $\widetilde{N}$ for a PN argument N, along with the set newly added set of transitions ($\widehat{T}$) and places ($\widehat{\Pi}$).

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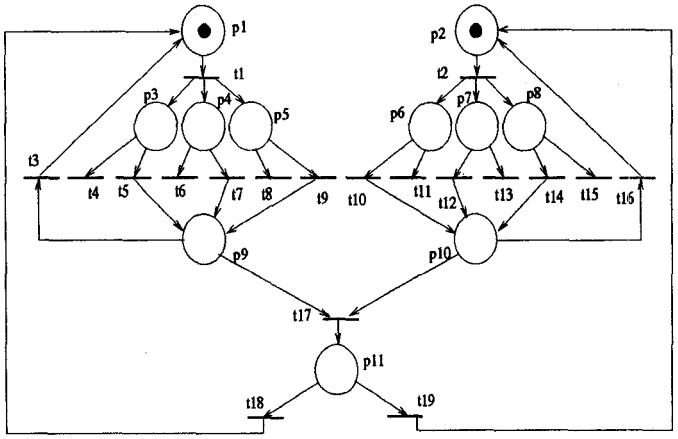
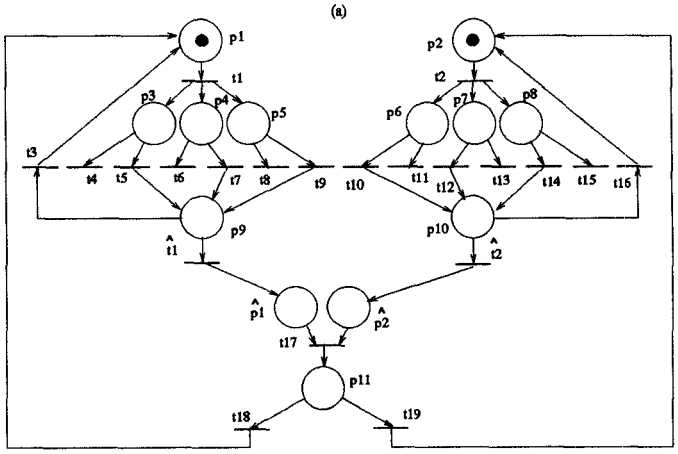


(a)



(b)

Figure 2: An illustration of the procedure $(\widetilde{N}, \widehat{T}, \widehat{\Pi}) =$ *convert_to_FCPN* (N), where N is the PN shown in (a), $\widetilde{N}$ is the PN shown in (b), $\widehat{T} = \{\widehat{t}_1, \widehat{t}_2\}$, and $\widehat{\Pi} = \{\widehat{p}_1, \widehat{p}_2\}$.

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firing string in N ($\widetilde{N}$) to that of a corresponding firing string in $\widetilde{N}$ (N). Theorems 3.1 and 3.2 together imply $L(\widetilde{N}, \widetilde{\mathbf{m}}^0) \mid_T = L(N, \mathbf{m}^0)$. For this reason, we refer to the FCPN $\widetilde{N}$ as the *free-choice equivalent* of the PN N.

Theorem 3.1 *Let* $(\widetilde{N}, \widehat{T}, \widehat{\Pi})$ = convert_to_FCPN (N), *where* $N = (\Pi, T, \Phi, \mathbf{m}^0)$ *and* $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ *are two PNs. If* $\mathbf{m}^0 \rightarrow \sigma \rightarrow \mathbf{m}^1$ *in* N *then* $\exists\ \widetilde{\sigma} \in \widetilde{T}^*$, $\exists\ \widetilde{\mathbf{m}}^1 \in \mathcal{N}^{card(\widetilde{\Pi})}$, *such that in* $\widetilde{N}$ *(i)* $\widetilde{\mathbf{m}}^0 \rightarrow \widetilde{\sigma} \rightarrow \widetilde{\mathbf{m}}^1$, *(ii)* $\widetilde{\mathbf{m}}^0 = \beta(\mathbf{m}^0)$ *and* $\widetilde{\mathbf{m}}^1 = \beta(\mathbf{m}^1)$, *and (iii)* $\widetilde{\sigma} \mid_T = \sigma$.

Theorem 3.2 *Let* $(\widetilde{N}, \widehat{T}, \widehat{\Pi})$ = convert_to_FCPN (N), *where* $N = (\Pi, T, \Phi, \mathbf{m}^0)$ *and* $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ *are two PNs. If* $\widetilde{\mathbf{m}}^0 \rightarrow \widetilde{\sigma}_1 \rightarrow \widetilde{\mathbf{m}}^1$ *in* $\widetilde{N}$ *then* $\mathbf{m}^0 \rightarrow \sigma_1 \rightarrow \mathbf{m}^1$ *in* N, *where* $\mathbf{m}^0 = \alpha(\widetilde{\mathbf{m}}^0)$, $\mathbf{m}^1 = \alpha(\widetilde{\mathbf{m}}^1)$, $\sigma_1 = \widetilde{\sigma}_1 \mid_T$, *and* $\forall p \in \Pi, \forall \widetilde{\mathbf{m}} \in \mathcal{N}^{card(\widetilde{\Pi})}, \alpha(\widetilde{\mathbf{m}})(p) = \widetilde{\mathbf{m}}(p) + \sum_{\widehat{p} \in ((p^\bullet)^\bullet_{\widetilde{N}})_{\widetilde{N}} \cap \widehat{\Pi}} \widetilde{\mathbf{m}}(\widehat{p})$.

The proof of theorems 3.1 and 3.2 are skipped for brevity. These results can be established by an induction argument over the length of the firing string σ, or $\widetilde{\sigma}_1 \mid_T$. Using theorems 2.2, 3.1 and 3.2 we show that there exists a supervisory policy that enforces liveness in the PN N if and only if there exists a corresponding policy for the PN $\widetilde{N}$.

Theorem 3.3 *Let* $(\widetilde{N}, \widehat{T}, \widehat{\Pi})$ = convert_to_FCPN (N), *where* $N = (\Pi, T, \Phi, \mathbf{m}^0)$ *and* $\widetilde{N} = (\widetilde{\Pi}, \widetilde{T}, \widetilde{\Phi}, \widetilde{\mathbf{m}}^0)$ *are two PNs. There exists a supervisory policy that enforces liveness in the PN* N *if and only if there exists a corresponding policy for the PN* $\widetilde{N}$.

Proof: (Only if part) If there exists a supervisory policy that enforces liveness in N, from theorem 2.2 we infer that in N, $\exists \mathbf{m}^1, \mathbf{m}^2 \in \mathcal{N}^{card(\Pi)}$, $\exists \sigma_1, \sigma_2 \in T^*$, such that (i) $\mathbf{m}^0 \rightarrow \sigma_1 \rightarrow \mathbf{m}^1 \rightarrow \sigma_2 \rightarrow \mathbf{m}^2$, (ii) $\mathbf{m}^2 \geq \mathbf{m}^1$, and (iii) every transition in T appears at least once in σ_2. From theorem 3.1 we infer that in $\widetilde{N}$, $\exists \widetilde{\mathbf{m}}^1, \widetilde{\mathbf{m}}^2 \in \mathcal{N}^{card(\widetilde{\Pi})}$, $\exists \widetilde{\sigma}_1, \widetilde{\sigma}_2 \in \widetilde{T}^*$, such that (i) $\widetilde{\mathbf{m}}^0 \rightarrow \widetilde{\sigma}_1 \rightarrow \widetilde{\mathbf{m}}^1 \rightarrow \widetilde{\sigma}_2 \rightarrow \widetilde{\mathbf{m}}^2$, (ii) $\forall\ \widehat{p} \in \widehat{\Pi}$, $\widetilde{\mathbf{m}}^0(\widehat{p}) = \widetilde{\mathbf{m}}^1(\widehat{p}) = \widetilde{\mathbf{m}}^2(\widehat{p}) = 0$, (iii) $\forall\ p \in \Pi$, $\widetilde{\mathbf{m}}^0(p) = \mathbf{m}^0(p)$, $\widetilde{\mathbf{m}}^1(p) = \mathbf{m}^1(p)$, $\widetilde{\mathbf{m}}^2(p) = \mathbf{m}^2(p)$, and (iv) $\widetilde{\sigma}_1 \mid_T = \sigma_1$ and $\widetilde{\sigma}_2 \mid_T = \sigma_2$. Since $\mathbf{m}^2 \geq \mathbf{m}^1$, we infer $\widetilde{\mathbf{m}}^2 \geq \widetilde{\mathbf{m}}^1$. Also, since all transitions in T appear at least once in σ_2, we conclude all transitions in T appear at least once in $\widetilde{\sigma}_2$. Additionally, from the construction we infer all transitions in $\widehat{T}$ must appear at least once in $\widetilde{\sigma}_2$ also. This is because, $\forall\ \widehat{t} \in \widehat{T}$, $\exists\ t \in T$, such that $card(\widehat{t}^\bullet \cap {}^\bullet t) = 1$. Using this observation, by applying theorem 2.2 to the FCPN $\widetilde{N}$ we conclude there exists a supervisory policy that enforces liveness in $\widetilde{N}$.

(If part) If there exists a supervisory policy that enforces liveness in $\widetilde{N}$, from theorem 2.2 we infer that in $\widetilde{N}$, $\exists \widetilde{\mathbf{m}}^1, \widetilde{\mathbf{m}}^2 \in \mathcal{N}^{card(\widetilde{\Pi})}$, $\exists \widetilde{\sigma}_1, \widetilde{\sigma}_2 \in \widetilde{T}^*$, such that (i) $\widetilde{\mathbf{m}}^0 \rightarrow \widetilde{\sigma}_1 \rightarrow \widetilde{\mathbf{m}}^1 \rightarrow \widetilde{\sigma}_2 \rightarrow \widetilde{\mathbf{m}}^2$, (ii) $\widetilde{\mathbf{m}}^2 \geq \widetilde{\mathbf{m}}^1$, and (iii) every transition in $\widetilde{T}$ appears at least once in $\widetilde{\sigma}_2$. From theorem 3.2 we infer that in N, $\exists \mathbf{m}^1, \mathbf{m}^2 \in \mathcal{N}^{card(\Pi)}$, $\exists \sigma_1, \sigma_2 \in T^*$, such that $\mathbf{m}^0 \rightarrow \sigma_1 \rightarrow \mathbf{m}^1 \rightarrow \sigma_2 \rightarrow \mathbf{m}^2$, where $\mathbf{m}^0 = \alpha(\widetilde{\mathbf{m}}^0)$, $\mathbf{m}^1 = \alpha(\widetilde{\mathbf{m}}^1)$, $\mathbf{m}^2 = \alpha(\widetilde{\mathbf{m}}^2)$, $\sigma_1 = \widetilde{\sigma}_1 \mid_T$ and $\sigma_2 = \widetilde{\sigma}_2 \mid_T$, where $\alpha : \mathcal{N}^{card(\widetilde{\Pi})} \rightarrow \mathcal{N}^{card(\Pi)}$ is defined in the statement of theorem 3.2. Since $\widetilde{\mathbf{m}}^2 \geq \widetilde{\mathbf{m}}^1$, we infer $\mathbf{m}^2 \geq \mathbf{m}^1$. Also, since every transition in $\widetilde{T}$ appears at least once in $\widetilde{\sigma}_2$ and $\widetilde{T} = T \cup \widehat{T}$, from the fact that $\sigma_2 = \widetilde{\sigma}_2 \mid_T$, we infer every transition in T appears at least once in σ_2. Using this observation, from theorem 2.2 we infer there is a supervisory policy that enforces liveness in N.

♣

Theorem 2.1 from reference [5] characterizes supervisory policies that enforce liveness in FCPNs. Reference [5] also introduces a class of FCPNs called *Independent, Increasing FCPNs* (II-FCPNs) for which a supervisory policy that enforces liveness is readily available. The FCPN $\widetilde{N}$ shown in figure 2(b) is an II-FCPN. This FCPN can be made live via supervision using a supervisory policy $\widetilde{\mathcal{P}}$ that simultaneously enforces the following inequalities (i) $\mathbf{m}(p_1) + \mathbf{m}(p_3) + \mathbf{m}(p_4) + \mathbf{m}(p_5) + \mathbf{m}(p_9) \geq 1$, and (ii) $\mathbf{m}(p_2) + \mathbf{m}(p_6) + \mathbf{m}(p_7) + \mathbf{m}(p_8) + \mathbf{m}(p_{10}) \geq 1$ (cf. section 4.1, [5]). From theorem 3.3 we infer there exists a supervisory policy that enforces liveness in the PN shown in figure 2(a). It can be shown that the supervisory policy of enforcing the above two inequalities also enforces liveness in the PN of figure 2(a).

We surmise that in addition to the class of II-FCPNs, there are many other classes of FCPNs for which a supervisory policy that enforces liveness is readily available. These classes of FCPNS are yet to be discovered. We envision the following scenario, the PN N is converted to a Free-Choice equivalent $\widetilde{N}$. If the FCPN $\widetilde{N}$ belongs to a class for which a policy is readily available, we seek to convert the policy that enforces liveness in $\widetilde{N}$ into a corresponding policy for

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N. However, for theorem 3.3 to be useful for a PN N that is not an FCPN, we need an effective procedure that converts a supervisory policy that enforces liveness in an (equivalent) FCPN $\widetilde{N}$ into a corresponding policy for the original PN N. For the particular case when $\widetilde{N}$ is an II-FCPN we conjecture the supervisory policy that enforces liveness in $\widetilde{N}$ also enforces liveness N. An investigation into this conjecture is suggested as a topic of future research.

4 Conclusions

In this paper we presented a procedure, that is similar to the one in appendix I of reference [1], that converts an arbitrary Petri net (PN) N into an equivalent Free-Choice Petri net (FCPN) $\widetilde{N}$. We show there is a supervisory policy that enforces liveness in the PN N if and only if there is a corresponding policy for the FCPN $\widetilde{N}$. With the characterization of supervisory policies that enforce liveness in FCPNs (cf. [5]), together with the identification of families of FCPNs for which a supervisory policy that enforces liveness can be constructed, it is hoped that these results will provide the much needed insight into the process of synthesizing supervisory policies that enforce liveness in arbitrary PNs.

References

[1] F. Bacelli, S. Foss, and B. Gaujal. Free choice nets, an algebraic approach. *IEEE Trans. on Automatic Control*, 12(41):1751–1778, December 1996.

[2] J.L. Peterson. *Petri net theory and the modeling of systems.* Prentice-Hall, Englewood Cliffs, NJ, 1981.

[3] W. Reisig. *Petri Nets.* Springer-Verlag, Berlin, 1985.

[4] R.S. Sreenivas. Enforcing liveness via supervisory control in discrete event dynamic systems modeled by completely controlled Petri nets. In *WODES-96: International Workshop on Discrete Event Systems*, pages 296–301, August 1996. University of Edinburgh, UK (IEE Press).

[5] R.S. Sreenivas. On commoner's liveness theorem and supervisory policies that enforce liveness in free-choice petri nets. *Systems & Control Letters*, 1997. To appear.

[6] R.S. Sreenivas. On the existence of supervisory policies that enforce liveness in discrete event dynamic systems modeled by controlled Petri nets. *IEEE Trans. on Automatic Control*, 42(7):928–945, July 1997.

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