An Application of Independent, Increasing, Free-Choice Petri Nets to the Synthesis of Policies that Enforce Liveness in Arbitrary Petri Nets*

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Abstract—The class of Independent, Increasing, Free-Choice Petri nets (II-FCPNs) was introduced in (Sreenivas, 1997c), where it is shown that any II-FCPN can be made live via supervision using a readily available policy. In a live Petri net (PN) (Peterson, 1981). Petri Net Theory and Modeling of systems. Prentice-Hall, Englewood Cliffs, N.J., Reissig (1985). Petri Nets. Springer, Berlin), it is possible to fire any transition from every reachable marking, although not necessarily immediately. In this paper we identify a class of PNs, where every transition is controllable, that are not necessarily II-FCPNs, that can be made live via supervision using a readily available policy constructed from the policy that enforces liveness in an II-FCPN. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction
We consider Petri net (PN) (Peterson, 1981; Reissig, 1985; Murata, 1989) models of discrete event dynamic systems (DEDS), where every transition is controllable, and we concern ourselves with the property of liveness in the PN models of DEDS. A PN is said to be live if it is possible to fire any transition from every reachable marking, although not necessarily immediately. In general, investigating the existence, and the synthesis of such policies can be computationally expensive (Sreenivas, 1997d). However, in this paper we show that the free-choice equivalent (Sreenivas, 1997b) of the original PN belongs to a restricted class called independent, increasing, free-choice petri nets (II-FCPNs) (Sreenivas, 1997c), there is a readily available supervisory policy that enforces liveness in the original PN.

Section 2 contains the notational preliminaries. Section (2) presents the main results of this paper, and in Section 4 we conclude with suggested future research topics.

2. Notations and definitions
A Petri net (PN) $N = (\Pi, T, \Phi, m^0)$ is an ordered 4-tuple, where $\Pi = \{p_1, p_2, \ldots, p_n\}$ is a set of n places, $T = \{t_1, t_2, \ldots, t_m\}$ is a set of m transitions, $\Phi \subseteq (\Pi \times T) \cup (T \times \Pi)$ is a set of arcs, $m^0: \Pi \rightarrow \mathbb{N}$ is the initial marking function (or the initial marking), and $\mathbb{N}$ is the set of nonnegative integers. The state of a PN is the marking $m: \Pi \rightarrow \mathbb{N}$ that identifies the number of tokens in each place. A PN $N$ is strongly connected if the reflexive, transitive closure of $\Phi$ equals the set $(\Pi \cup T) \times (T \cup \Pi)$. For a given marking $m$, a transition $t \in T$ is said to be enabled if $\forall \pi \in \Phi$, $m(\pi) \geq 1$, where $\Phi = \{\pi \mid \pi = (x, y), x \in \Phi\}$. The set of enabled transitions is denoted by the symbol $T_m(m)$. An enabled transition $t \in T_m(m)$ can fire, which changes the marking $m$ to $m'$ according to the equation $m'(p) = m(p) - \text{card}(\pi^+ (t)) + \text{card}(\pi^- (t))$, (1)

where $\pi^+ (t) = \{\pi \mid \pi = (x, y), y \in \Phi\}$, and the symbol $\text{card}(\pi)$ is used to denote the cardinality of the set argument. This notation is also used to denote the predecessor or successor set of a set of places or transitions. A collection of places $P \subseteq \Pi$ is said to be a siphon (trap) if $\forall \pi \in \Phi$, $\pi^+(\pi) \in P^*$. A trap (siphon) $P^*$ is said to be minimal if $\exists P^* \subset P$ such that $P^* = P^*(P^* \cap P)$

A string of transitions $\sigma$ is said to be a valid firing string starting from the marking $m$ if after the firing of each transition in the sequence denoted by $\sigma$ the marking remains non-negative. The set of reachable markings from $m^0$, denoted by $\Theta(N, m^0)$, is the set of markings generated by all valid firing strings starting with marking $m^0$ in the PN $N$. At a marking $m^0$, if the firing of a valid firing string $\sigma$ results in a marking $m^*$, we represent it as $m^1 \rightarrow \sigma \rightarrow m^*$. A transition $t \in T$ is live if $\exists m^1 \in \Theta(N, m^0)$, $\exists m^2 \in \Theta(N, m^0)$ such that $t \in T_m(m^1)$

For any $\sigma \in \Sigma^*$, and for any $T \subseteq T$, the projection of $\sigma$ onto the subset $T$, denoted by $\sigma|_T$, is the string of transitions in $T$ obtained from $\sigma$ by erasing all transitions that are not in $\tau$. While retaining the transition order in the rest of the string. A supervisory policy $\pi: \Sigma^* \rightarrow \{[0,1]^m\}$, is a total map that returns an $m$-dimensional binary vector for each reachable marking. The supervisory policy $\pi$ permits the firing of transition $t_i$ at marking $m$, only if $\pi(m)|_T \neq 1$. A string of transitions $\sigma = t_1 t_2 \ldots t_k$ is said to be a valid firing string under the supervision of $\pi$ starting from the marking $m$, if (ii) the transition $t_i$ is enabled at the marking $m$, $\pi(m)|_T \neq 1$, and (ii) for $t \in \Pi, 2, \ldots, k - 1$, the firing of the transition $t_i$ produces a marking $m'$ at which the transition $t_i$ is enabled and $\pi(m')|_T \neq 1$. A set of reachable markings under the supervision of $\pi$ in $N$ from the initial marking $m^0$ is denoted by $\Theta(N, m^0, \pi)$. A transition $t_i$ is live under the supervision of $\pi$ if $\forall m^1 \in \Theta(N, m^0), m^2 \in \Theta(N, m^0, \pi)$ such that $t \in T_m(m)$ and $\pi(m)|_T \neq 1$. A supervisory policy $\pi$ enforces liveness if all transitions in $N$ are live under $\pi$. Sreenivas (1997d) presents a necessary and sufficient condition for the existence of a supervisory policy that enforces liveness in an arbitrary PN. This condition is testable in general, but it is testable if the PN is bounded, or, if each transition in the PN can be prevented from firing by the supervisory policy. Testing the existence of a supervisory policy that enforces liveness in an arbitrary, bounded PN is at least PSPACE-complete (Sreenivas, 1997d). The class of PNs we consider in this paper are not necessarily testable, but we assume each transition can be prevented from firing by the supervisory policy.
A PN \( N = (\Pi, T, \Phi, m^0) \) is a free-choice PN (FCPN) if
\( \forall \eta \in \Pi, \text{card}(\eta^*) > 1 \Rightarrow (\eta^*) = [\eta] \). The class of policies that
enforce liveness in FCPNs is characterized in (Sreenivasan, 1997c).
This reference also introduced a new class of FCPNs called
independent, increasing, free-choice Petri nets (ICFPNs), and it is
shown that any IFCPN can be made live via supervision
using a readily available supervisory policy. We now present
the definition of IFCPNs.

For an arbitrary FCPN \( \hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{m}^0) \), an independent,
increasing supervisory (IS) subnet is an IFCPN \( \hat{S} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{m}^0) \) where
\( \exists \hat{T} \subseteq \hat{T}^* \) such that (i) \( \hat{T} = \hat{T}^* \subseteq \hat{\Pi}, \) (ii) \( \exists \hat{t} \in \hat{T}, \) such that
\( \text{card}(\hat{t}^*) > 1, \) (iii) \( \hat{S} \) is the induced subnet by the set of places \( \hat{T} \),
and (iv) there is at least one token in \( \hat{T} \) at the initial marking. The set of transitions \( \hat{T} \) is called the critical transition subset of the IFCPN
\( \hat{S} \). The IFCPN
induced by the set of places \( \hat{T} \), and set of transitions \( \hat{T} \) is referred to as the
strongly connected component of \( \hat{S} \).

An independent, increasing FCPN (IFCPN) is an FCPN where
\( \forall t \in T, \text{card}(t^*) > 1 \) at the initial marking, or (ii) is an IFCPN. In (Sreenivasan, 1997c) it is
shown that any supervisory policy that maintains at least one
mark on the input place-set \( \hat{T} \), for example, of each IFCPN enforces
liveness.

Any arbitrary PN \( N = (\Pi, T, \Phi, m^0) \) can be converted to an
equivalent FCPN \( \hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{m}^0) \) with the addition of a few
extra places and transitions (Hack, 1974). The conversion
procedure is outlined in Fig. 1 of Sreenivasan (1997b). Additionally,
in this reference it is shown that there exists a supervisory policy
that enforces liveness in the PN \( N \) if and only if there exists a
common policy for the FCPN \( \hat{N} \). The PN \( N \) shown in
Fig. 2a of (Sreenivasan, 1997b) is not an FCPN. To see this note
that \( \eta^* \neq \eta^* \) if \( \eta \neq \eta \).

The PN shown in Fig. 2b of this reference is an FCPN obtained by using the
procedure convert to FCPN (N) in Section 4.1 of Sreenivasan (1997c) it is
shown that the FCPN \( \hat{N} \) is an IFCPN. The repetition of these
observations are skipped for brevity.

Since the PN \( N \) and the FCPN \( \hat{N} \) share common places and
transitions it is necessary to define the following sets to
eliminate any confusion. The symbol \( \eta^* \) denotes the set
\( \{ t \in T | t \neq \phi, \eta \} \); and the symbol \( \eta^* \) denotes the set
\( \{ t \in T | t \neq \phi, \eta \} \). The symbols \( \eta^* \), \( \eta^* \), \( \eta^* \), \( \eta^* \), \( \eta^* \), \( \eta^* \), \( \eta^* \), \( \eta^* \) are
defined accordingly.

In the following section, we consider the family of PNs
\( N = (\Pi, T, \Phi, m^0) \) that yield an IFCPN when the procedure
convert to FCPN is applied to \( N \). That is, we consider the
family of PNs \( N \) such that \( \hat{N} = \hat{N} \) is the
supervisory policy that maintains at least one token in the input place-set of
each IFCPN \( \hat{S} \) can be modified to yield a policy that enforces
liveness in the PN \( N \).

3. Main Results

Let \( N = (\Pi, T, \Phi, m^0) \) be PN, that is not necessarily an
IFCPN, such that \( \hat{N} = (\hat{\Pi}, \hat{T}, \hat{\Phi}, \hat{m}^0) \) is an IFCPN. We
demonstrate that there exists a readily constructible supervisory
policy that enforces liveness in \( N \). In Lemmas 1 and 2, we make a few
observations on the influence of the supervisory policy \( \hat{N} \) that maintains
at least one token in the input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

Lemma 1. Let \( \hat{N} \) denote the supervisory policy that maintains at least one token in the
input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

Lemma 2. Let \( \hat{N} \) denote the supervisory policy that maintains at least one token in the
input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

Suppose that \( \exists t \in T, f(t) \neq 0 \) at the marking \( m \),
then \( \sum \hat{m}^0 \) is a marking of \( N \).

Lemma 3. Let \( \hat{N} \) denote the supervisory policy that maintains at least one token in the
input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

Lemma 4. Let \( \hat{N} \) denote the supervisory policy that maintains at least one token in the
input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

Proof. Let \( \hat{N} \) denote the supervisory policy that maintains at least one token in the
input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

Theorem 1. Let \( \hat{N} \) denote the supervisory policy that maintains at least one token in the
input place-set of each IFCPN \( \hat{S} \) can be modified to yield a policy that
enforces liveness in the PN \( N \).

For the supervisory policy \( \hat{N} \) introduced above, the following
lemma relates the existence of a firing string under the supervision
of \( \hat{N} \) in \( N \) to that of a corresponding string in \( \hat{N} \) under the
supervision of \( \hat{N} \).
Lemma 5. If $m^0 \rightarrow \sigma_1 \rightarrow m^1$ under the supervision of $\mathcal{P}$ in $N$ then $\beta(m^0) \rightarrow \sigma_1 \rightarrow \beta(m^1)$ under the supervision of $\mathcal{P}$ in $\bar{N}$, where $\sigma_{1|1} = \sigma_1$, and the supervisory policies $\mathcal{P}$ and $\bar{\mathcal{P}}$ are as defined earlier.

Proof. This result is established by an induction argument over the length of $\sigma_1$. The base case is established by letting $\sigma_1$ equal the empty string. As the induction hypothesis let $m^0 \rightarrow \sigma_1 \rightarrow m^1$ under the supervision of $\mathcal{P}$ in $N$ and $\beta(m^0) \rightarrow \sigma_1 \rightarrow \beta(m^1)$ under the supervision of $\bar{\mathcal{P}}$ in $\bar{N}$ such that $\sigma_{1|1} = \sigma_1$ when the length of $\sigma_1 \leq n$ for some $n \in \mathbb{N}$.

As the induction step let $m^0 \rightarrow \sigma_1 \rightarrow m^1 \rightarrow t_i \rightarrow m^1$ under the supervision of $\mathcal{P}$ in $N$. Since $m^1 \rightarrow t_i \rightarrow m^1$ under the supervision of $\mathcal{P}$ in $N$, we infer the existence of $\sigma_2$ such that $\beta(m^1) \rightarrow \sigma_2 \rightarrow m$ under the supervision of $\bar{\mathcal{P}}$ in $\bar{N}$ where $\sigma_{2|1} = t_i$. From Lemma 3, we infer the existence of $\sigma_3$ such that $\beta(m^1) \rightarrow \sigma_3 \rightarrow \beta(\mathcal{M}_{\mathcal{P}})$ where $\sigma_{3|1} = \sigma_{2|1} = t_i$. From the construction of $\bar{N}$ we infer $\beta(\mathcal{M}_{\bar{\mathcal{P}}}) = \beta(m^1)$. Hence the result.

The following theorem establishes the fact that the supervisory policy $\mathcal{P}$ enforces liveness in $N$.

Theorem 1. Let $\mathcal{P}$ be the supervisory policy for the PN $N$ as defined earlier, then $\mathcal{P}$ enforces liveness in $N$.

Proof. From Theorem 2 (Sreenivas, 1997c), we know the supervisory policy $\mathcal{P}$ enforces liveness in the II-FCPN $\bar{N}$. Suppose $m^0 \rightarrow \sigma_1 \rightarrow m^1$ under the supervision of $\mathcal{P}$ in $N$, then by Lemma 5 we infer $\beta(m^0) \rightarrow \sigma_1 \rightarrow \beta(m^1)$ under the supervision of $\mathcal{P}$ in $\bar{N}$. Since $\mathcal{P}$ enforces liveness in $N$, $\forall t_i \in T$; $\exists \sigma_2 \in \mathcal{P}$, such that $\beta(m^0) \rightarrow \sigma_2 \rightarrow m$ under the supervision of $\mathcal{P}$ in $\bar{N}$. From Lemma 4 we infer $\exists (\beta(m^0)(x = m^0) \rightarrow \sigma_{2|1} \rightarrow \beta(\mathcal{M}_{\mathcal{P}}))$ under the supervision of $\mathcal{P}$ in $N$. Therefore, $\mathcal{P}$ enforces liveness in $N$.

4. Conclusions

We considered Petri nets (PNs) whose free-choice equivalent is an independent, increasing, free-choice Petri net (II-FCPN) (cf. Section 4.1, Sreenivas, 1997c). For the class of non-live, II-FCPNs there exists a readily available supervisory policy that enforces liveness. In a live PN, it is possible to fire any transition from every reachable marking, although not necessarily immediately. In this paper we showed the readily available policy that enforces liveness in the equivalent II-FCPN can be effectively converted to a similar policy for the original PN that is not necessarily an II-FCPN. As a future research direction we suggest investigations into other classes of free-choice equivalent PNs for which there is a readily available supervisory policy that enforces liveness, and an investigation into procedures that convert these policies into a corresponding policy for the original PN. We also suggest identification of structural conditions of the original net that imply the converted FCPN will be an II-FCPN as an additional future research direction.

References


